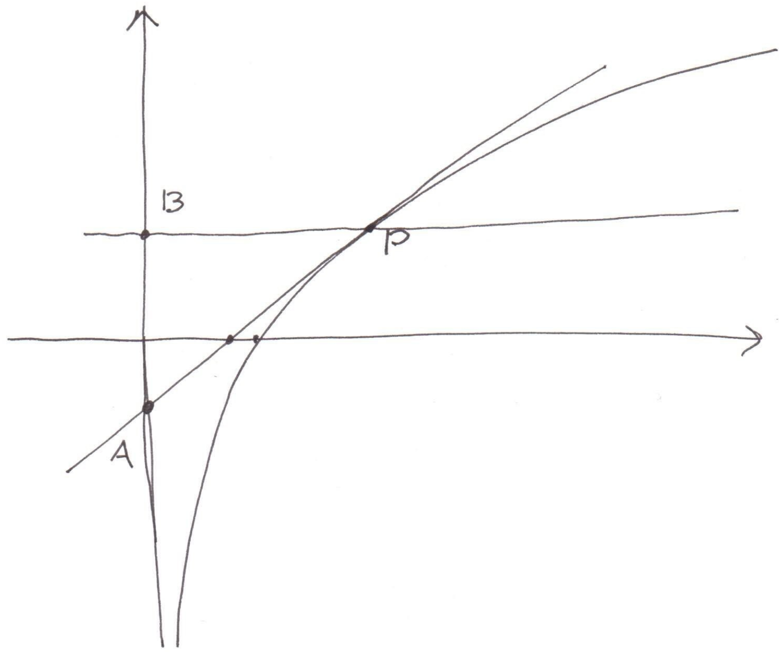
1) $\quad$ (1



$f(x) = \log(x)$ $\qquad G_f = \{(x, \log x) : x \in \mathcal{D}\}$

$\mathcal{D} = (0, +\infty)$

$P(\alpha, \log\alpha) \in G_f \Rightarrow$

$y - \log\alpha = f'(\alpha)(x - \alpha)$

$t_p: \boxed{y = \frac{1}{\alpha}x - 1 + \log\alpha}$

$A = t_p \cap \{x = 0\} \Rightarrow A(0, \log\alpha - 1)$

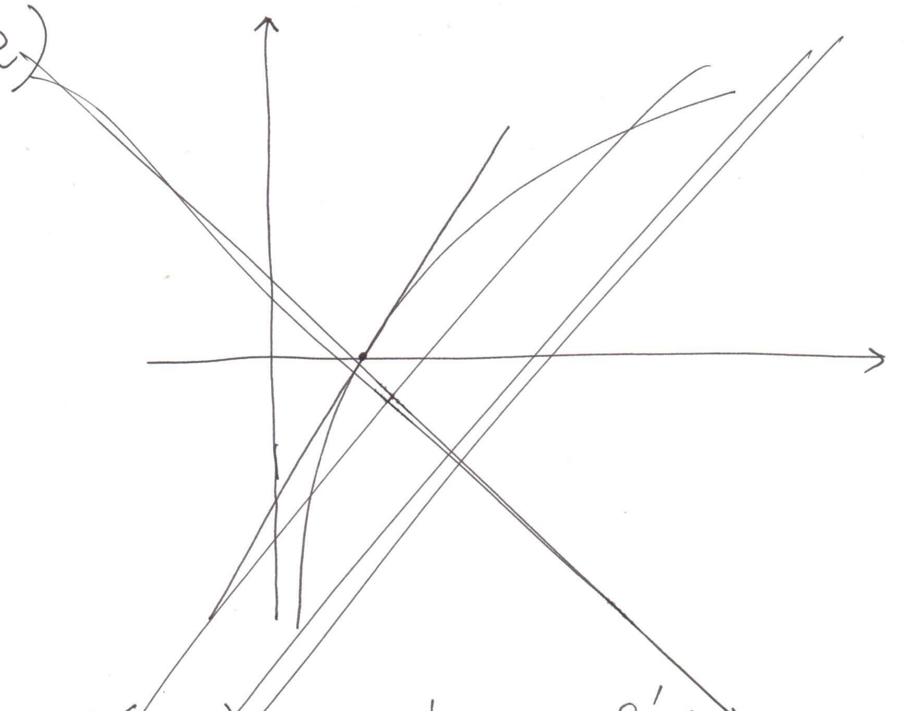
$s_p : y = \log\alpha \Rightarrow B = s_p \cap \{x = 0\}$

$B(0, \log\alpha)$

$AB = |\log x - \log x + 1| = 1$

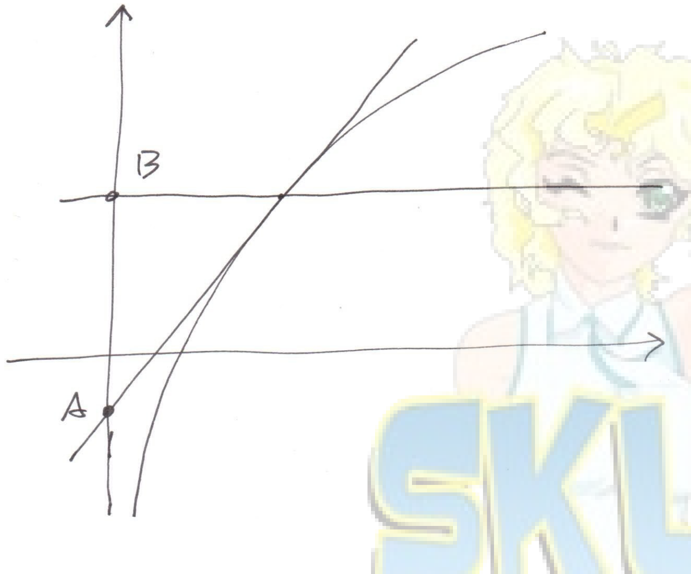
AB costante

2)



$S(1,0) \Rightarrow \tan \delta = f'(1) = 1$

$\Rightarrow \delta = 45^\circ$



$g(x) = \log_a x$

$g'(x) = \dfrac{1}{x \log a}$ $\qquad P(\alpha, \log_a \alpha)$

$t_P: \; y - \log_a \alpha = \dfrac{1}{\alpha \log a}(x - \alpha)$

$A\left(0, \log_a \alpha - \dfrac{1}{\log a}\right)$

$B(0, \log_a \alpha)$

$|AB| = \dfrac{1}{\log a}$ dipende da a

2)

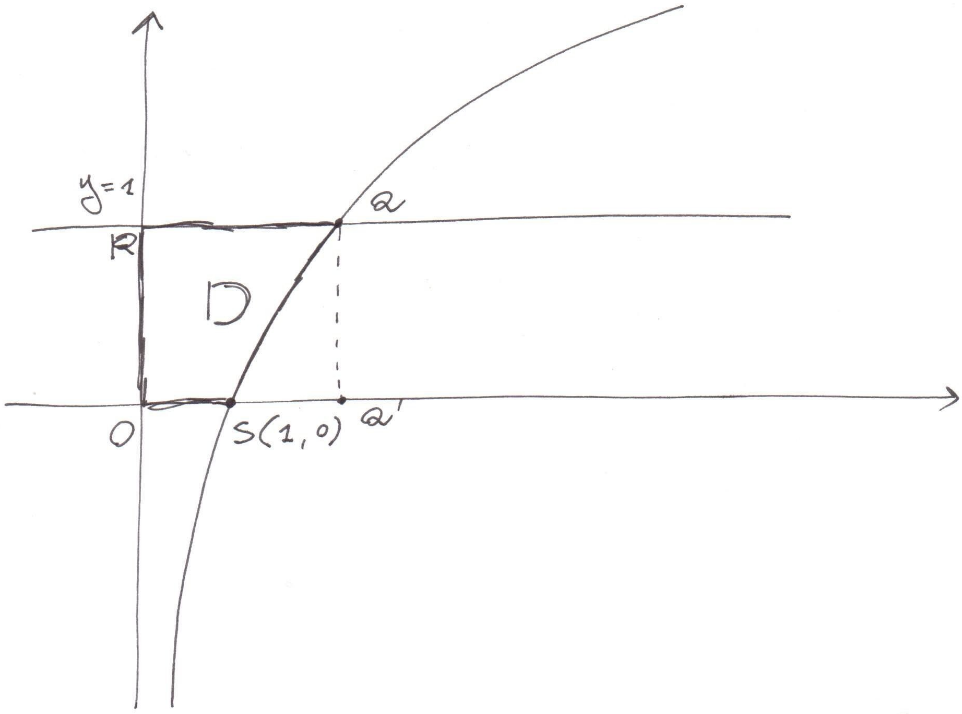
$S(1,0) \Rightarrow \tg \delta = g'(1) = \dfrac{1}{\log a}$

$\Rightarrow \delta = 45° \Rightarrow 1 = \dfrac{1}{\log a}$

$\Rightarrow \log a = 1 \Rightarrow a = e$

$\delta = 135° \Rightarrow -1 = \dfrac{1}{\log a}$

$\Rightarrow \log a = -1 \Rightarrow a = \dfrac{1}{e}$



$$Q = \{y = 1\} \cap \{y = \log x\}$$

$$\Rightarrow Q(e, 1)$$

$$Q'(e, 0)$$

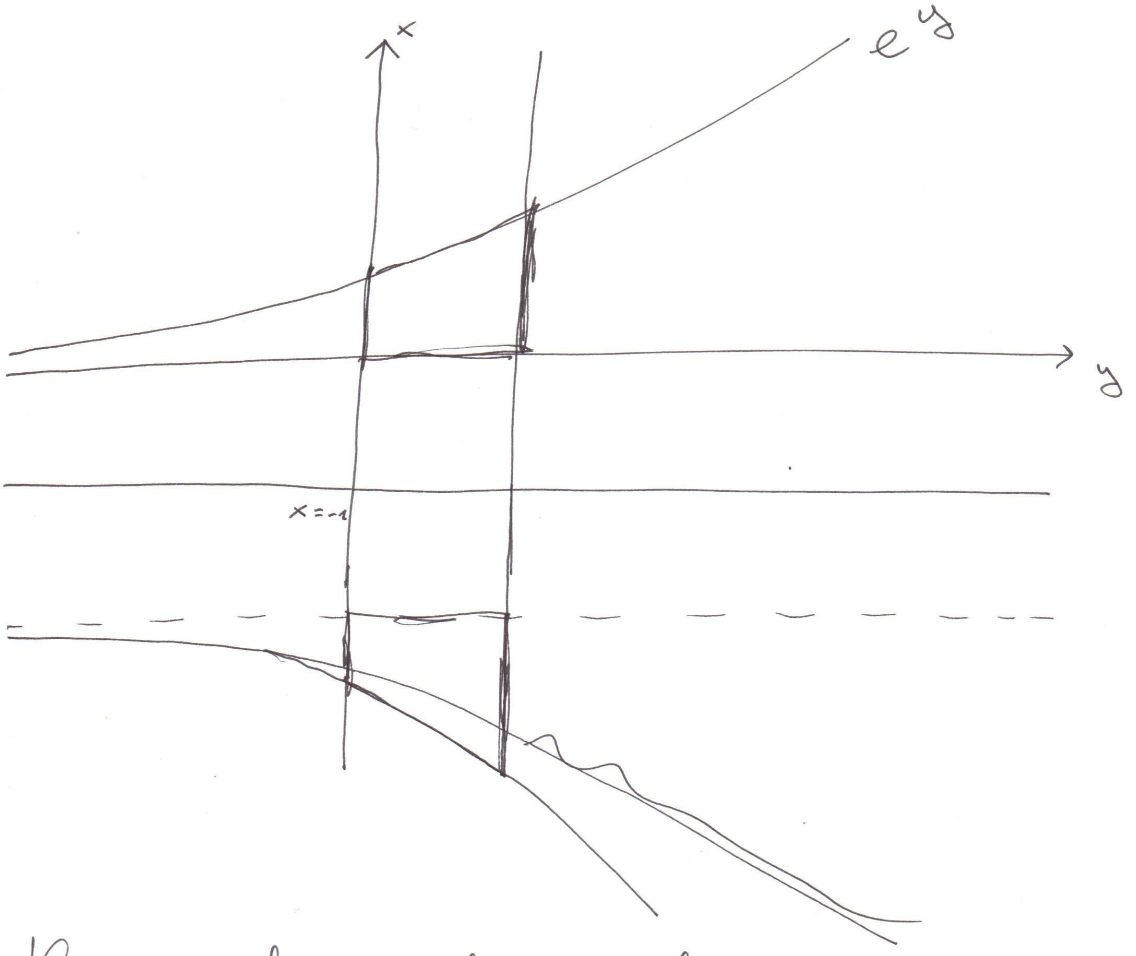
$$\Rightarrow D = A_{ORQQ'} - \int_1^e \log x \, dx$$

$$= e - \int_1^e \log x \, dx =$$

$$= e - \left[x \cdot \log x \Big|_1^e - \int_1^e dx \right] =$$

$$= e - e + 0 + e - 1 = \boxed{e - 1}$$

Sia $y = \log x \Rightarrow$
$x = e^y$



Il centro di rotazione è sull'asse $x = -1$
$\Rightarrow$ traslo in alto di $+1$ per avere asse di rotazione $x = 0$

$$\Rightarrow \quad h(y) = e^y + 1$$

$$V_D = \int_0^1 h(y)^2 \, dy - V_{cie}$$

dove V_{cie} è il volume del cilindro di raggio 1 e altezza 1. Allora

$$V_D = \pi \int_0^1 (e^y + 1)^2 \, dy - \pi =$$

$$= \pi \left[\int_0^1 (e^{2y} + 2e^y + 1) \, dy - 1 \right]$$

$$= \pi \left[\frac{1}{2} e^{2y} \Big|_0^1 + 2 e^y \Big|_0^1 + y \Big|_0^1 - 1 \right]$$

$$= \pi \left[\frac{1}{2} e^2 - \frac{1}{2} + 2e - 2 + \cancel{1} - \cancel{1} \right]$$

$$= \pi \left[\frac{e^2}{2} + 2e - \frac{5}{2} \right]$$