P 1 Scient. tec. L1

$$f(x) = \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}\right)e^{-x}$$

$n \in \mathbb{N}$, $x \in \mathbb{R}$

1. $$f'(x) = \left(1 + x + \dots + \frac{n x^{n-1}}{n!}\right)e^{-x}$$

$$- \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}\right)e^{-x}$$

$$= \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!}\right)e^{-x}$$

$$- \left(1 + x + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}\right)e^{-x}$$

$$= -\frac{x^n}{n!}e^{-x}$$

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2. $f'(x) = 0 \iff \underline{x = 0}$

Se n è pari $\Rightarrow f'(x) \leq 0 \quad \forall x$

Se n è dispari $\Rightarrow f'(x) > 0, \ x < 0$

$f'(x) < 0, \ x > 0$

Allora se n pari $\Rightarrow f$ sempre decrescente e non ha max e min.

Se n dispari allora

f cresce su $(-\infty, 0)$

f decresce su $(0, +\infty)$

f ha un max in $x = 0$ e

$\underline{f(0) = 1}$ $\quad M(0, 1)$

Ora

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x^n}{n!} \cdot e^{-x} \quad (\text{f.i. } \infty \cdot 0)$$

$$= \lim_{x \to +\infty} \frac{x^n}{n! \cdot e^x} = \quad (\text{con de l'Hopital } n \text{ volte})$$

$$= \lim_{x \to +\infty} \frac{n!}{n!\, e^x} = 0^+$$

$$\lim_{x \to -\infty} f(x) = \infty \quad (\text{il segno dipende da } n)$$

Allora se n dispari

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \left(\frac{x^n}{n!} \right) e^{-x}$$ 3

posto $t = -x$

$$= \lim_{t \to +\infty} \frac{(-1)^n t^n}{n!} e^{t} =$$

$$= -\frac{1}{n!} \lim t^n e^t = -\infty$$

Allora se n dispari
$M(0,1)$ massimo assoluto
e quindi $f(x) \leq 1$

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(3) $n = 2$

$$\boxed{f(x) = \left(1 + x + \frac{x^2}{2}\right) e^{-x}}$$

$$= \frac{e^{-x}}{2} \left(x^2 + 2x + 2 \right) =$$

$$= \frac{e^{-x}}{2} \left[(x+1)^2 + 1 \right]$$

f risulta sempre positiva
e mai nulla. Inoltre

$x = 0 \Rightarrow f(0) = 1$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2 e^{-x}}{2} = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x^2 e^{-x}}{2} = 0^+$$

$$m = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x e^{-x}}{2} = -\infty$$

$\Rightarrow$ $y = 0$ asintoto orizzontale
a destra
no asintoti vert e obl.

$$f'(x) = -\frac{x^2}{2} e^{-x} \leq 0 \quad \forall x$$

Non ci sono massimi
e la f decresce.

$$f''(x) = -\frac{1}{2}\left[2x e^{-x} - x^2 e^{-x}\right] =$$

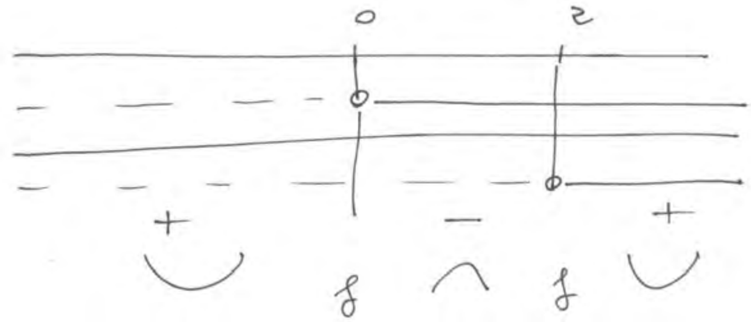
$$= \frac{x e^{-x}}{2}\left[x - 2\right] \geq 0$$

$x \geq 0$

$e^{-x} > 0 \quad \forall x$

$x - 2 \geq 0 \qquad x \geq 2$

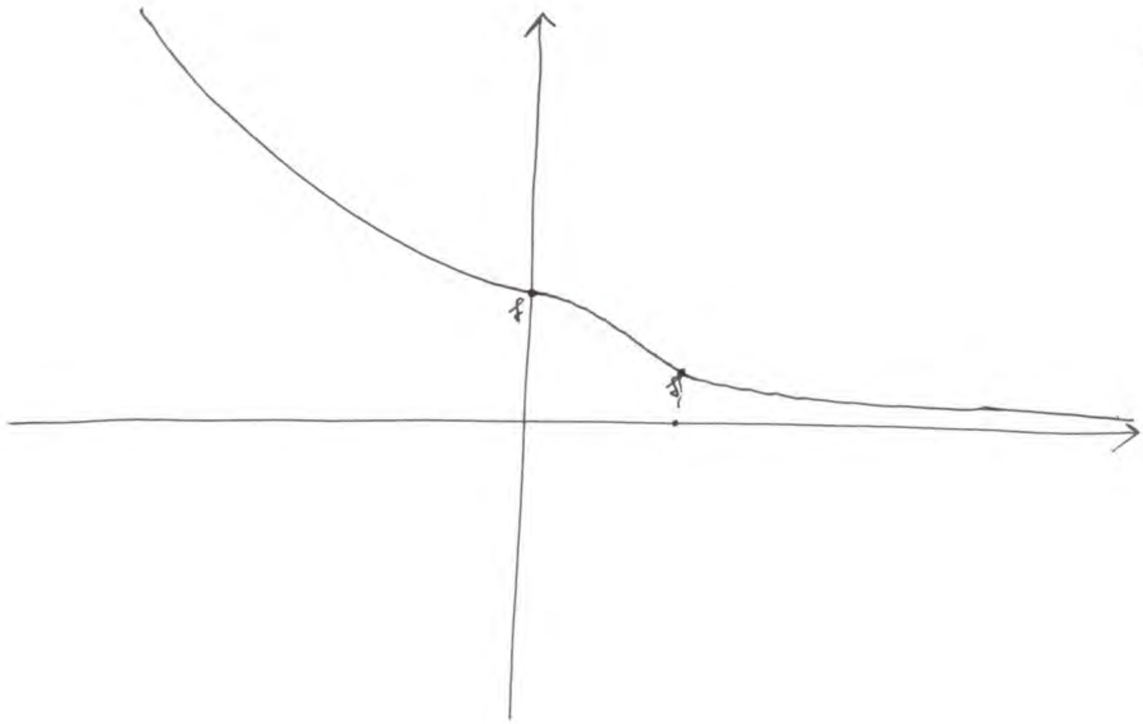
$\Rightarrow$



2 flessi :

$x = 0 \qquad f(0) = 1$

$x = 2 \qquad f(2) = \frac{5}{e^2}$



(4) $\int_0^2 g(x)\,dx =$ | Tale int rappresenta l'area sottesa da g e limitata dalle rette $x=0$ e $x=2$

$$= \int_0^2 \frac{e^{-x}}{2}\left[x^2+2x+1\right]dx =$$

$$= -\frac{e^{-x}}{2}\left(x^2+2x+2\right)\Big|_0^2 + \int_0^2 \frac{e^{-x}}{2}(2x+2)\,dx$$

$$= -\frac{5}{e^2} + 1 + \left[-\frac{e^{-x}}{2}(2x+2)\Big|_0^2 + \int_0^2 \frac{e^{-x}}{2}\cdot 2\,dx\right]$$

$$= -\frac{5}{e^2} + 1 - \frac{3}{e^2} + 1 - e^{-x}\Big|_0^2 = 2 - \frac{8}{e^2} - \frac{1}{e^2} + 1$$

$$= 3 - 9/e^2$$