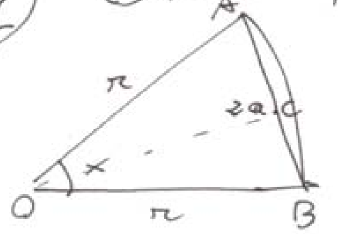
① ORD P1 $\quad AB = 2a$ 



$$AB^2 = r^2 + r^2 - 2r^2 \cos x$$

(Carnot)

$$= 2r^2(1 - \cos x) = 4a^2$$

$$\Rightarrow \quad \underline{a^2 = \frac{r^2}{2}(1 - \cos x)} \;\Rightarrow\; a = \frac{r}{\sqrt{2}}\sqrt{(1 - \cos x)}$$

$$A_{\widehat{OAB}} = \frac{x r^2}{2}$$

per l'altezza del triangolo

$$OC = \sqrt{r^2 - a^2} = \sqrt{r^2 - \frac{r^2}{2} + \frac{r^2}{2}\cos x}$$

$$= \sqrt{\frac{r^2}{2}} \cdot \sqrt{1 + \cos x} = \frac{r}{\sqrt{2}}\sqrt{1 + \cos x}$$

$$\Rightarrow A_{A\hat{O}B} \overset{\frac{1}{2} OC \cdot 2a =}{=} \frac{1}{\cancel{2}} \cdot \frac{r}{\sqrt{2}}\sqrt{1 + \cos x} \cdot \frac{\cancel{2}r}{\sqrt{2}}\sqrt{1 - \cos x}$$

$$= \frac{r^2}{2}\sqrt{(1 - \cos^2 x)} = \frac{r^2}{2}\sin x$$

$$\Rightarrow \quad S(x) = \frac{x r^2}{2} - \frac{r^2}{2}\sin x =$$

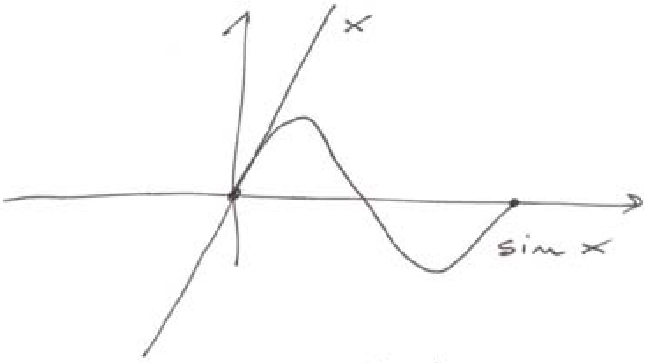
$$= \frac{r^2}{2}(x - \sin x)$$

② Studio di $S(x)$ su $[0, 2\pi]$

$\underline{S(0) = 0}$ $\quad S(x) > 0$

$\Rightarrow \quad x - \sin x > 0$

Ma




$\Rightarrow \quad S(x) > 0 \quad \forall \; x \in [0, 2\pi]$

Non ci sono limiti.

$$S'(x) = \frac{1}{2}\left(1 - \cos x\right) \geq 0 \quad \forall x$$

$\cos x \leq 1$

$\Rightarrow$ S sempre crescente

Poiché S ristretta a $[0, 2\pi]$

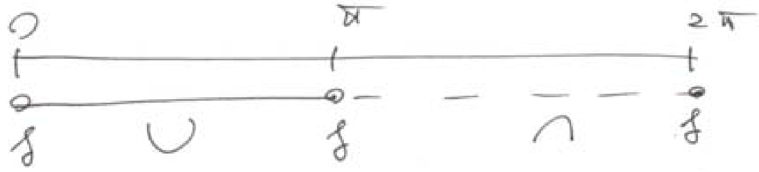
$\Rightarrow \; x = 0$ min assoluto $m(0,0)$

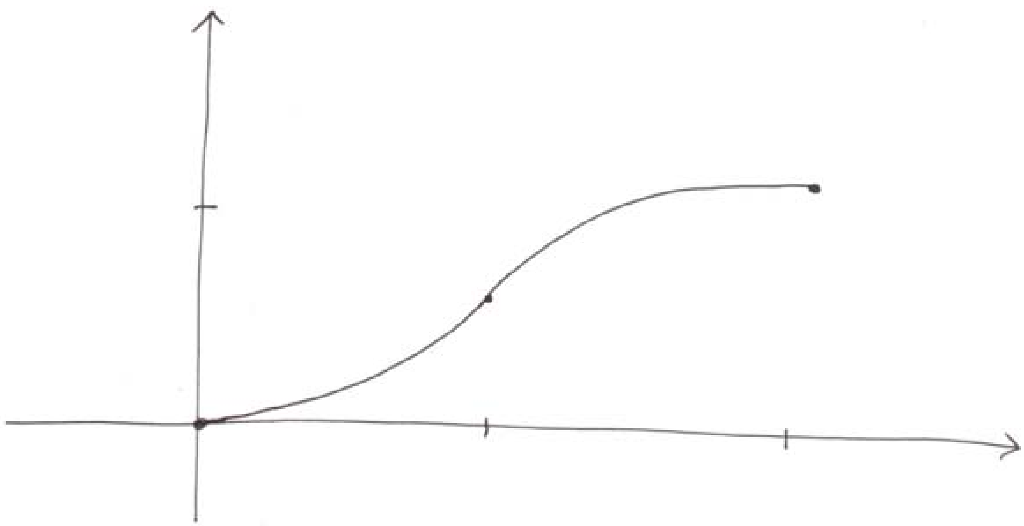
$\quad\;\; x = 2\pi$ Max " $M(2\pi, \pi)$

$$S''(x) = \frac{\sin x}{2} \geq 0$$

$$\Rightarrow \quad \sin x \geq 0$$

$$\Rightarrow \quad 0 \leq x \leq \pi \,, \quad x = 2\pi$$



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③ $A_{\widehat{AOB}} = \frac{1}{2} x r^2 = 100\, m^2$

$$\Rightarrow \quad \underline{x = \frac{200}{r^2}}$$

$P_{\widehat{AOB}} = 2r + l$ $\quad$ $l$ lunghezza dell'arco.

$$\frac{2\pi r}{2\pi} = \frac{l}{x} \Rightarrow \underline{l = x r}$$

$$\Rightarrow P(r) = 2r + xr =$$
$$= 2r + \frac{200}{r} = 2\left(r + \frac{100}{r}\right)$$

$$P'(r) = 2\left(1 - \frac{100}{r^2}\right) \geq 0$$

$$\frac{r^2 - 100}{r^2} \geq 0 \Rightarrow r^2 - 100 \geq 0$$

$$\cancel{-10 \leq r \leq 10} \qquad \underline{r \geq 10}$$

$$\cancel{\Rightarrow 0 \leq r \leq 10 \ (r \text{ è pos.}}$$

$$\cancel{r = 10 \text{ massimo}}$$

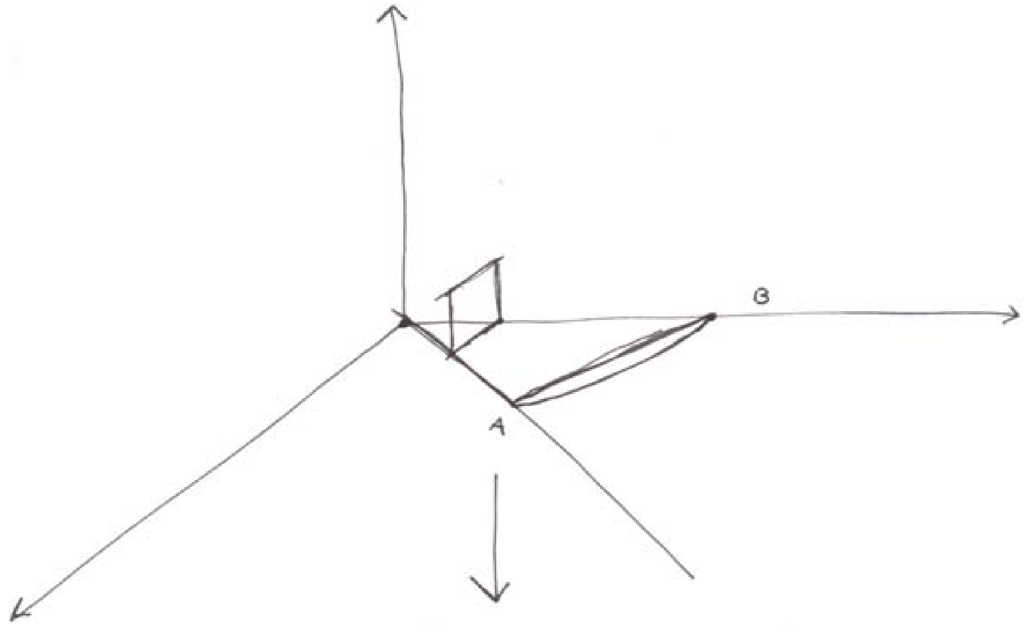
$$\Rightarrow r = 10 \text{ minimo!}$$

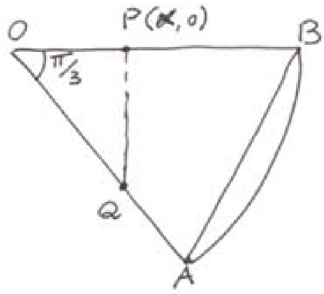
$$\underline{P(10) = 40}$$

$\underline{x = 2}$ se $\alpha$ è il valore di $x$ in gradi

$$\frac{\alpha}{360°} = \frac{x}{2\pi} \Rightarrow \alpha = \frac{360°}{\pi} =$$
$$\underline{\simeq 114{,}59°}$$

4) $r = 2 \qquad x = \frac{\pi}{3}$




$0 \le \alpha \le 2$ (perché $r = 2$)

Se $P(\alpha, 0)$ allora

$Q = \{x = \alpha\} \cap \{OA\}$

$OA : \; y = \left(tg\,\frac{\pi}{3}\right) x = \sqrt{3}\, x$

$\underline{Q(\alpha, \sqrt{3}\,\alpha)}$

$$PQ = \ell(\alpha) = \sqrt{3} \cdot \alpha$$

lato del quadrato

$$\Rightarrow \quad W = \int_0^2 \ell^2(\alpha)\, d\alpha =$$

$$= \int_0^2 3\alpha^2 \, d\alpha = \alpha^3 \Big|_0^2 = 8$$

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